Statistical Properties of Wideband MIMO Mobile-to-Mobile Channels

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Abstract—A three-dimensional (3-D) theoretical model for wideband multiple-input multiple-output (MIMO) mobile-to-mobile (M-to-M) channels is presented. Based on this model, the statistical properties of wideband MIMO M-to-M channels are derived. In particular, the space-time-frequency correlation function, the power space-delay spectral density, and the envelope level crossing rate are derived for a 3-D non-isotropic scattering environment. Finally, to validate the theoretical derivations, some simulation results are presented and compared with measured data.

I. INTRODUCTION

Mobile-to-mobile (M-to-M) communications play an important role in mobile ad-hoc wireless networks, intelligent transportation systems, and relay-based cellular networks. M-to-M communication systems are equipped with low elevation antennas and have both the transmitter (Tx) and the receiver (Rx) in motion. To successfully design M-to-M systems, it is necessary to have a detailed knowledge of the outdoor multipath fading channel and its statistical properties. The statistical properties of M-to-M channels are quite different from conventional fixed-to-mobile (F-to-M) cellular land mobile radio channels [1]. Akki and Haber [2] proposed a reference model for single-input single-output (SISO) M-to-M Rayleigh fading channels. The reference models for narrowband multiple-input multiple-output (MIMO) M-to-M channels have been proposed in [3], [4]. The previously reported models are two-dimensional (2-D) and accurate only for certain environments, e.g., rural areas. For urban environments where the Tx and Rx antenna arrays are often located in close proximity to and lower than surrounding buildings, the tree-dimensional (3-D) models are more appropriate. Hence, we have recently proposed the three-dimensional (3-D) reference models for narrowband and wideband MIMO M-to-M multipath fading channels [5], [6] as well as the simulation models for wideband MIMO M-to-M channels [7].

This paper uses the 3-D reference model in [6] to derive the statistical properties of wideband MIMO M-to-M channels. The 3-D reference model generates the input delay-spread function as a superposition of the line-of-sight (LoS), single-bounced and double-bounced rays. The parametric nature of the model makes it adaptable to a variety of propagation environments, i.e., outdoor micro- and macro-cells. Based on this model, the space-time-frequency correlation function (stf-cf), the power space-delay spectral density (psds), and the envelope level crossing rate (LCR) are derived for a 3-D non-isotropic scattering environment. Finally, to verify the theoretical derivations, some simulation results are presented and compared with measured data in [8], and [9].

The remainder of the paper is organized as follows. Section II presents the geometrical “concentric-cylinders” model and the 3-D parametric reference model for wideband MIMO M-to-M channels. Section III derives the stf-cf, the psds, and the LCR for a 3-D non-isotropic scattering environment. Section IV presents some simulation results to verify theoretical derivations. Finally, Section V provides some concluding remarks.

II. A 3-D THEORETICAL MODEL FOR WIDEBAND MIMO MOBILE-TO-MOBILE CHANNELS

This section describes a 3-D theoretical model for wideband MIMO M-to-M multipath fading channels. We consider a MIMO communication system with $L_t$ transmit and $L_r$ receive omnidirectional antenna elements. It is assumed that both the Tx and Rx are in motion and equipped with low elevation antennas. The radio propagation in outdoor micro- and macro-cells is characterized by 3-D wide sense stationary uncorrelated scattering (WSSUS) with either line-of-sight (LoS) or non-line-of-sight (NLoS) conditions between the Tx and Rx. The MIMO channel is described by an $L_r \times L_t$ matrix $\mathbf{H}(t, \tau) = [h_{ij}(t, \tau)]_{L_r \times L_t}$ of the input delay-spread functions.

Fig. 1 shows the 3-D “concentric-cylinders” geometrical model with $L_t = L_r = 2$ antenna elements. The “concentric-cylinders” model defines four cylinders, two around the Tx and another two around the Rx, as shown in Fig. 1. Around the Tx, $M$ fixed omnidirectional scatterers occupy a volume between cylinders of radii $R_{t1}$ and $R_{t2}$. It is assumed that the $M$ scatterers lie on $L$ cylindric surfaces of radii $R_{t1} \leq R_t(l) \leq R_{t2}$, where $1 \leq l \leq L$. The $l^{th}$ cylindric surface contains $M^{(l)}$ fixed omnidirectional scatterers, and the $(m, l)^{th}$ transmit scatterer is denoted by $S_T^{(m, l)}$. Similarly, around the Rx, $N$ fixed omnidirectional scatterers occupy a volume between cylinders of radii $R_{r1}$ and $R_{r2}$. It is assumed that $N$ scatterers lie on $K$ cylindric surfaces of radii $R_{r1} \leq R_r(k) \leq R_{r2}$, where $1 \leq k \leq K$. The $k^{th}$ cylindric surface contains $N^{(k)}$ fixed omnidirectional scatterers, and the $(n, k)^{th}$ receive scatterer is

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denoted by $S_{R}^{(n,k)}$. The parameters in Fig. 1 are defined in Table 1.

It is assumed that the radii $R_{t_2}$ and $R_{r_2}$ are much smaller than the distance $D$, i.e., max{ $R_{t_2}, R_{r_2}$ } $\ll$ $D$ (local scattering condition for outdoor micro- and macro-cells). Furthermore, it is assumed that the distance $D$ is smaller than $4R_{t_1}R_{r_1}/(\lambda((L_{t} - 1)/(L_{r} - 1))$ (channel does not experience keyhole behavior [10]), where $\lambda$ denotes the carrier wavelength. Finally, it is assumed that $d_{T}$ and $d_{R}$ are much smaller than the radii $R_{t_1}$ and $R_{r_1}$, i.e., max{ $d_{T}, d_{R}$ } $\ll$ min{ $R_{t_1}, R_{r_1}$ }.

From the 3-D geometrical model, we observe that waves from the Tx antenna elements traverse directly to the Rx antenna elements or they are single- or double-bounced before arriving at the Rx antenna elements. Hence, the input delay-spread function of the link $A_{T}^{(p)} - A_{R}^{(q)}$ can be written as a superposition of the LoS, single-bounced, and double-bounced rays [6], i.e., $h_{pq}(t, \tau)$ is equal to

$$h_{pq}^{SBT}(t, \tau) + h_{pq}^{SBR}(t, \tau) + h_{pq}^{DB}(t, \tau) + h_{pq}^{LS}(t, \tau).$$

(1)

To simplify further derivations, we will use the time-variant transfer function (i.e., the Fourier transformation of the input delay-spread function), which can be written as

$$T_{pq}(t, f) = \mathcal{F}\{h_{pq}(t, \tau)\} = T_{pq}^{SBT}(t, f) + T_{pq}^{SBR}(t, f) + T_{pq}^{DB}(t, f) + T_{pq}^{LS}(t, f).$$

(2)

The single- and double-bounced components of the time-variant transfer function are, respectively,

$$T_{pq}^{SBT}(t, f) = \sqrt{\sqrt{M}} \lim_{M \rightarrow \infty} \sum_{l=1}^{L} \sum_{m=1}^{M^{(t)}} \xi_{m,l} g_{m,l}(t) e^{-j2\pi f \tau_{m,l}},$$

(3)

$$T_{pq}^{SBR}(t, f) = \sqrt{\sqrt{R}} \lim_{N \rightarrow \infty} \sum_{k=1}^{K} \sum_{n=1}^{N^{(k)}} \xi_{n,k} g_{n,k}(t) e^{-j2\pi f \tau_{n,k}},$$

(4)

$$T_{pq}^{DB}(t, f) =$$

$$\sqrt{\sqrt{M}} \lim_{M \rightarrow \infty} \sum_{l=1}^{L} \sum_{m=1}^{M^{(t)}} \xi_{m,l} g_{m,l}(t) e^{-j2\pi f \tau_{m,l,n,k}},$$

(5)

where parameters $\xi_{m,l}$, $\xi_{n,k}$, $\xi_{m,l,n,k}$, $\tau_{m,l}$, $\tau_{n,k}$, and $\tau_{m,l,n,k}$ denote amplitudes of the multipath components and time delays, respectively. Functions $g_{m,l}(t)$, $g_{n,k}(t)$, and $g_{m,l,n,k}(t)$ are defined as follows

$$g_{m,l}(t) = e^{-j \frac{2\pi}{\lambda} \left(\xi_{m,l} T_{t} + \phi_{m,l} + n_{m,l} \right)} + j \theta_{m,l},$$

(6)

$$g_{n,k}(t) = e^{-j \frac{2\pi}{\lambda} \left(\xi_{n,k} T_{r} + \phi_{n,k} + n_{n,k} \right)} + j \theta_{n,k},$$

(7)

$$g_{m,l,n,k}(t) = e^{-j \frac{2\pi}{\lambda} \left(\xi_{m,l,n,k} T_{t} + \phi_{m,l,n,k} + n_{m,l,n,k} \right)} + j \theta_{m,l,n,k},$$

(8)

where $f_{\text{max}} = v_{r}/\lambda$ and $f_{\text{max}} = v_{r}/\lambda$ are the maximum Doppler frequencies associated with the Tx and Rx, respectively, and $\lambda$ is the carrier wavelength. The amplitudes $\xi_{m,l}$, $\xi_{n,k}$, and $\xi_{m,l,n,k}$ are approximated as

$$\xi_{m,l} \approx \frac{\Omega_{pq}}{\sqrt{M(K_{pq} + 1)}},$$

(9)

$$\xi_{n,k} \approx \frac{\Omega_{pq}}{\sqrt{N(K_{pq} + 1)}},$$

(10)

$$\xi_{m,l,n,k} \approx \frac{\Omega_{pq}}{\sqrt{MN(K_{pq} + 1)}} \left(1 - \frac{\gamma R_{t}^{(l)} + R_{r}^{(k)}}{2D} \right),$$

(11)

where $\Omega_{pq} = D^{-\gamma/2} \sqrt{P_{pq} A_{t} A_{r} P_{pq}}$, $P_{pq}$ is the power transmitted through the subchannel $A_{T}^{(p)} - A_{R}^{(q)}$, $K_{pq}$ denotes the Rice factor (ratio of LoS to scatter received power) of the subchannel $A_{T}^{(p)} - A_{R}^{(q)}$, and $\gamma$ is the path loss exponent. The time delays $\tau_{m,l}$ and $\tau_{m,l,n,k}$ are defined as the travel times

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**Table 1**

**Definition of parameters in Figure 1.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$D$</td>
<td>The distance between the centers of the Tx and Rx cylinders.</td>
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<tr>
<td>$R_{t}^{(l)}$, $R_{r}^{(k)}$</td>
<td>The radius of the $l$th Tx and $k$th Rx cylinder, respectively.</td>
</tr>
<tr>
<td>$d_{T}$, $d_{R}$</td>
<td>The spacing between antenna elements at the Tx and Rx, respectively.</td>
</tr>
<tr>
<td>$\theta_{t}$, $\theta_{r}$</td>
<td>The orientation of the Tx and Rx antenna array in the x-y plane (relative to the y-axis), respectively.</td>
</tr>
<tr>
<td>$\Psi_{t}$, $\Psi_{r}$</td>
<td>The elevation of the Tx’s and Rx’s antenna array relative to the x-y plane, respectively.</td>
</tr>
<tr>
<td>$\tau_{t}$, $\tau_{r}$</td>
<td>The velocities of the Tx and Rx, respectively.</td>
</tr>
<tr>
<td>$d_{T}(m,l)$, $d_{T}(k,n)$</td>
<td>The azimuth angles of departure (AAD) of the waves that impinge on the scatterers $s_{m,l}$ and $s_{k,n}$, respectively.</td>
</tr>
<tr>
<td>$d_{R}(m,l)$, $d_{R}(k,n)$</td>
<td>The azimuth angles of arrival (AADA) of the waves scattered from $s_{m,l}$ and $s_{k,n}$, respectively.</td>
</tr>
<tr>
<td>$h_{t}$, $h_{r}$</td>
<td>The AADA of the LoS paths.</td>
</tr>
<tr>
<td>$\xi_{m,l}$, $\xi_{n,k}$, $\xi_{m,l,n,k}$</td>
<td>The distances $d_{A}(m,l)^{T}$, $d_{A}(m,l)^{R}$, $d_{A}(m,l)^{T} - d_{A}(m,l)^{R}$, $d_{A}(m,l)^{T}$, $d_{A}(m,l)^{R}$, $d_{A}(m,l)^{T} - d_{A}(m,l)^{R}$, $d_{A}(m,l)^{T}$, $d_{A}(m,l)^{R}$, $d_{A}(m,l)^{T} - d_{A}(m,l)^{R}$, respectively.</td>
</tr>
<tr>
<td>$\tau_{m,l}$</td>
<td>The distances $d_{A}(m,l)^{T}$, $d_{A}(m,l)^{R}$, $d_{A}(m,l)^{T} - d_{A}(m,l)^{R}$, respectively.</td>
</tr>
</tbody>
</table>
of the waves scattered from the scatterers $S^{(m,l)}_T$ and $S^{(n,k)}_R$, i.e. $\tau_{m,l} = D/c_0 + R^{(l)}_R (1 - \cos \alpha^{(m,l)}_T)/c_0 \cos \beta^{(m,l)}_T$ and $\tau_{n,k} = D/c_0 + R^{(k)}_R (1 + \cos \alpha^{(n,k)}_R)/c_0 \cos \beta^{(n,k)}_R$, respectively, where $c_0$ is the speed of light. Finally, the time delay $\tau_{m,l,n,k}$ is defined as the travel time of the wave impinged on the scatterer $S^{(m,l)}_T$ and scattered from the scatterer $S^{(n,k)}_R$, i.e. $\tau_{m,l,n,k} = D/c_0 + R^{(l)}_R \cos \alpha^{(n,k)}_T/c_0 \cos \beta^{(n,k)}_R + R^{(k)}_R \cos \alpha^{(n,k)}_R/c_0 \cos \beta^{(n,k)}_R$, respectively, where $c_0$ is the speed of light. Finally, the time delay $\tau_{m,l,n,k}$ is defined as the travel time of the wave impinged on the scatterer $S^{(m,l)}_T$ and scattered from the scatterer $S^{(n,k)}_R$, i.e. $\tau_{m,l,n,k} = D/c_0 + R^{(l)}_R (1 - \cos \alpha^{(m,l)}_T)/c_0 \cos \beta^{(m,l)}_T + R^{(k)}_R (1 + \cos \alpha^{(n,k)}_R)/c_0 \cos \beta^{(n,k)}_R$, respectively, where $c_0$ is the speed of light.

III. STATISTICAL PROPERTIES OF WIDEBAND MIMO M-TO-M CHANNELS

This section derives the stf-cf, the psds, and the LCR for a 3-D non-isotropic scattering environment. The normalized stf-cf between two time-variant transfer functions $T_{pq}(t, f)$ and $T_{pq'}(t, f)$ is defined as

$$R_{pq,pq'}(\Delta t, \Delta f) = \frac{E[T_{pq}(t, f) T_{pq'}^*(t + \Delta t, f + \Delta f)]}{E[|T_{pq}(t, f)|^2] E[|T_{pq'}(t, f)|^2]}$$

where $(\cdot)^*$ denotes complex conjugate operation, $E[\cdot]$ is the statistical expectation operator, $p, p' \in \{1, \ldots, L_t\}$, and $q, q' \in \{1, \ldots, L_r\}$. Since the number of local scatterers in the reference model is infinite, the discrete AAoDs, $\alpha^{(m,l)}_T$, $\beta^{(m,l)}_T$, $\alpha^{(n,k)}_R$, $\beta^{(n,k)}_R$, and radii $R^{(l)}_R$ and $R^{(k)}_R$ can be replaced with continuous random variables $\alpha_T, \beta_T, \alpha_R, \beta_R, R_t, \text{ and } R_r$. To characterize the random azimuth angles $\alpha_T$ and $\alpha_R$, we use the von Mises probability density function (pdf) defined as $[11] f(\theta) = \exp \left[k \cos(\theta - \mu)/2\pi I_0(k)\right]$, where $\theta \in [-\pi, \pi]$, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, $\mu \in [-\pi, \pi]$ is the mean angle at which the scatterers are distributed in the $x$ - $y$ plane, and $k$ controls the spread of scatterers around the mean. To characterize the random elevation angles $\beta_T$ and $\beta_R$, we use the pdf $[12] f(\phi) = \sin(\phi)/(4 \varphi_m) / (\varphi_m) |\varphi| \leq \varphi_m \leq \pi/2$ and $f(\phi) = 0$ otherwise. The parameter $\varphi_m$ is the maximum elevation angle and lies in the range $0 \leq \varphi_m \leq 20^\circ$ [13], [6]. To characterize the radii $R_t$ and $R_r$, we use the pdfs $[6] f(R_t) = 2R_t/(R_t^2 - R_{t1}^2)$ and $f(R_r) = 2R_r/(R_r^2 - R_{r1}^2)$, respectively. Using trigonometric transformations, the identity $\int_0^{\pi} \exp \{a \sin(c) + b \cos(c)\} dc = 2\pi I_0(\sqrt{a^2 + b^2})$ [14], eq. 3.338-4], and the results in [5], the stf-cfs of the single- and double-bounced components can be written as
where $x_{SBT} \approx j2\pi[(p-\tilde{p})d_{T_x}/\lambda + \Delta tf_{Bmax}\cos\gamma_T + \Delta tf_{Rmax}\cos\gamma_R + \Delta tf_{K}\cos\gamma_K + k_T\cos\mu_T, y_{SBT} \approx j2\pi[(p-\tilde{p})d_{T_y}/\lambda + (q-\tilde{q})d_{T_y}/\lambda + \Delta tf_{Bmax}\sin\gamma_T + \Delta tf_{Rmax}\sin\gamma_R + k_T\sin\mu_T, x_{SBR} \approx j2\pi[(q-\tilde{q})d_{T_x}/\lambda + \Delta tf_{Bmax}\cos\gamma_T + \Delta tf_{Rmax}\cos\gamma_R + \Delta tf_{K}\cos\gamma_K + k_T\cos\mu_T, y_{SBR} \approx j2\pi[(q-\tilde{q})d_{T_y}/\lambda + \Delta tf_{Bmax}\sin\gamma_T + \Delta tf_{Rmax}\sin\gamma_R + k_T\sin\mu_T$, and $\Delta f = \eta_T R \cos(2\pi\beta_T d_{T_x}/\lambda) \cos(2\pi\beta_{Rmax} d_{T_y}/\lambda) e^{-j2\pi\Delta f/c_0}$.}

Finally, the normalized stf-cf between two time-variant transfer functions $T_{pq}(t, f)$ and $T_{\tilde{pq}}(t, f)$ becomes a summation of the stf-cfs in (14) - (17).

The psds of the time-variant transfer function is the inverse Fourier transformation of the space-frequency correlation function $R_{pq,\tilde{pq}}(d_T, d_R, \Delta t = 0, \Delta f)$. Using the equality [14, eq. 6.677-3] and after extensive calculations, the psds of the single- and double-bounded components become

\[
P^{SBT}_{pq,\tilde{pq}}(d_T, d_R, \tau_{rel}) = \frac{\eta_T}{I_0(k_T)} \cos \left( \frac{2\pi}{\lambda} \beta_{Tm} d_T \right) \left( 1 - \left( \frac{4\beta_{Rm} d_R}{T_2} \right)^2 \right) e^{-j2\pi(p-\tilde{q})d_{T_x}} \times 
\frac{2}{R_{T^2} - R_{1^2}} \int_{R_{ta}}^{R_{tb}} \left( 1 - \frac{R_t}{R_D} \right) R_t e^{j2\pi c_0 R_t B_{SBT}(c_0\tau_{rel}/R_{ta}-1)} \cosh \left( 2jC_{SBT} \right) \frac{c_0 \tau_{rel}}{2R_{ta}} \left( 1 - \frac{c_0 \tau_{rel}}{2R_{2a}} \right) \frac{\tau_{rel}}{R_{ta}} dR_t,
\]

\[
P^{SBR}_{pq,\tilde{pq}}(d_T, d_R, \tau_{rel}) = \frac{\eta_R}{I_0(k_R)} \cos \left( \frac{2\pi}{\lambda} \beta_{Rm} d_R \right) \left( 1 - \left( \frac{4\beta_{Tm} d_T}{T_2} \right)^2 \right) e^{-j2\pi(p-\tilde{q})d_{T_y}} \times 
\frac{2}{R_{T^2} - R_{1^2}} \int_{R_{ta}}^{R_{tb}} \left( 1 - \frac{R_t}{R_D} \right) R_t e^{j2\pi c_0 R_t B_{SBR}(c_0\tau_{rel}/R_{ta}-1)} \cosh \left( 2jC_{SBR} \right) \frac{c_0 \tau_{rel}}{2R_{ta}} \left( 1 - \frac{c_0 \tau_{rel}}{2R_{2a}} \right) \frac{\tau_{rel}}{R_{ta}} dR_t,
\]

where $\cosh \left( \cdot \right) \) is the hyperbolic cosine, $\otimes$ denotes convolution, $\tau_{rel} = \tau - D/t_0, F_t = e^{2\pi R_t B_{DB}(c_0\tau_{rel}/R_{ta}-1)/c_0}, F_{r} = e^{2\pi R_r B_{DB}(c_0\tau_{rel}/R_{ra}-1)/c_0}, B_{DB} = [(p-\tilde{p})d_{T_x}/\lambda - jk_T\cos\mu_T] / \pi T_2, C_{DB} = [(q-\tilde{q})d_{T_y}/\lambda - jk_R\cos\mu_R] / \pi T_2, R_{ta} = c_0\tau_{rel}/2, R_{tb} = R_{ta}, R_{ra} = R_{ta}, R_{rb} = R_{tb}$. The integrals in (18) - (20) need to be numerically evaluated over all possible radii $R_t$ and $R_R$, respectively. For the range $0 \leq \tau_{rel} \leq 2R_{ta}/c_0$, integration limits are $R_{ta} = R_{ta1}, R_{tb} = R_{tb2}, R_{ra} = R_{tb1},$ and $R_{rb} = R_{rb2}$. On the other hand, when $2R_{ta}/c_0 \leq \tau_{rel} \leq 2R_{tb}/c_0$, integration limits are $R_{ta} = c_0\tau_{rel}/2, R_{tb} = R_{ta}, R_{ra} = c_0\tau_{rel}/2,$ and $R_{rb} = R_{tb}$. The psds of the LoS component is obtained by calculating the inverse Fourier transformation of the space-frequency correlation function in (17) and can be written as

\[
P^{LoS}_{pq,\tilde{pq}}(d_T, d_R, \tau_{rel}) = \sqrt{K_{pq} K_{\tilde{pq}}} e^{j2\pi(p-\tilde{q})d_{T_x}} \times 
\frac{2}{R_{T^2} - R_{1^2}} \int_{R_{ta}}^{R_{tb}} \left( 1 - \frac{R_t}{R_D} \right) R_t e^{j2\pi c_0 R_t B_{LoS}(c_0\tau_{rel}/R_{ta}-1)} \cosh \left( 2jC_{LoS} \right) \frac{c_0 \tau_{rel}}{2R_{ta}} \left( 1 - \frac{c_0 \tau_{rel}}{2R_{2a}} \right) \frac{\tau_{rel}}{R_{ta}} dR_t,
\]

where $\delta(\cdot)$ denotes the Dirac delta function. Finally, the psds $P_{pq,\tilde{pq}}(d_T, d_R, \tau_{rel})$ between two time-variant transfer functions $T_{pq}(t = 0, f)$ and $T_{\tilde{pq}}(t = 0, f)$ becomes a summation of the psds in equations (18) - (21).

The LCR at a specified level $R, L(R)$, is defined as the rate at which the signal envelope crosses level $R$ in the positive (or negative) going direction. When a LoS component is present, the LCR can be calculated as [15]

\[
L(R) = \frac{2R \sqrt{K + 1}}{\pi^{3/2}} \left( \frac{b_2}{b_0} - \frac{b_1}{b_0} \right) e^{-K/(K+1) R^2}
\]
\[
\times \int_0^{\pi/2} \cos \left( 2\sqrt{K(K+1)} R \cos \theta \right) \times \left[ e^{-\left( \chi \sin \theta \right)^2} + \sqrt{\pi} \chi \sin \theta \text{erf}(\chi \sin \theta) \right] d\theta, \quad (22)
\]

where \( \cosh(\cdot) \) is the hyperbolic cosine function, \( \text{erf}(\cdot) \) is the error function, and the parameter \( \chi \) is equal to \( \sqrt{K(b_1^2/(b_0 b_2) - b_1^2))} \), and \( K \) denotes the averaged Rice factor. Finally, parameters \( b_0, b_1, \) and \( b_2 \) are defined as \([16]\)

\[
b_0 \triangleq E[h_i(t, \tau = 0)^2] = E[h_i(t, \tau = 0)^2], \quad (23)
\]

\[
b_1 \triangleq E[h_i(t, 0)h_q(t, 0)] = E[h_q(t, 0)h_i(t, 0)], \quad (24)
\]

\[
b_2 \triangleq E[h_i(t, 0)^2] = E[h_q(t, 0)^2], \quad (25)
\]

where \( h_i(t, 0) \) and \( h_q(t, 0) \) denote the in-phase and quadrature component of the input delay-spread function \( h_0(0) = h_0^{SBT}(t, 0) + h_0^{SBR}(t, 0), \) \( h_0^{SBR}(t, 0) \) denotes the statistical expectation operator, and \( h_i(t, 0) \) and \( h_q(t, 0) \) denote the first derivative of \( h_i(t, 0) \) and \( h_q(t, 0) \) with respect to time \( t \). The parameters \( b_0, b_1, \) and \( b_2 \) are obtained by substituting (1) into (23), (24) and (25), respectively, and by using trigonometric transformations, and the equality \( \int_0^\pi e^{-i \theta} \sin \theta d\theta = 2\pi J_m(\sin \theta) \) \([14, \text{eq. 8.411}]\), where \( J_m(\cdot) \) is the \( m \)-th order Bessel function of the first kind. After extensive calculations, the parameters \( b_0, b_1, \) and \( b_2 \) become

\[
b_n = b_n^{SBT} + b_n^{SBR} + b_n^{DB}, \quad (26)
\]

where \( n \in \{0, 1, 2\} \), \( b_0^{SBT} = \eta_{SR}(K + 1) \), \( b_0^{DB} = \eta_{TR}/(K + 1) \), and the parameters \( b_1^{SBT} \), \( b_1^{DB} \), \( b_2^{SBT} \), and \( b_2^{DB} \) are, respectively,

\[
\begin{align*}
\eta_{TR} &= \pi^2 \cos \beta_{TR} - 2\pi f_{T\text{max}} \Delta T_{TR} \sin \gamma_{TR} \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\end{align*}
\]

\[
\begin{align*}
\eta_{TR} &= \frac{\pi^2 \cos \beta_{TR} - 2\pi f_{T\text{max}} \cos \mu_T(k_T/R)}{2} \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\end{align*}
\]

\[
\begin{align*}
b_1^{DB} &= \frac{\eta_{TR}}{K + 1} \left\{ \frac{\pi^2 \cos \beta_{TR} - 2\pi f_{T\text{max}} \cos \mu_T(k_T/R)}{2} \right\} \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\end{align*}
\]

\[
\begin{align*}
b_2^{SBT} &= \frac{2\pi^2 \eta_{TR}(K + 1)}{K + 1} \left\{ \frac{\pi^2 \cos \beta_{TR} - 2\pi f_{T\text{max}} \cos \mu_T(k_T/R)}{2} \right\} \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\end{align*}
\]

\[
\begin{align*}
b_2^{DB} &= \frac{2\pi^2 \eta_{TR}(K + 1)}{K + 1} \left\{ \frac{2\pi^2 \cos \beta_{TR} - 2\pi f_{T\text{max}} \cos \mu_T(k_T/R)}{2} \right\} \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\cos \mu_T(k_T/R) &= \frac{1}{2} \cos^2 \beta_{TR} - 4\beta_{TR}^2 \\
\end{align*}
\]

IV. SIMULATION RESULTS

In this section, we present some simulation results to verify the theoretical derivations. In Figs. 2 and 3, we plot the psds, \( P_{pq,pq}(d_T, d_R, \tau_m) \), for several MIMO systems. In these figures, we analyze the radio propagation in outdoor M-to-M micro- and macro-cells, assuming 3-D non-isotropic scattering \( (k_T = k_R = 9.4) \) for curves in Fig. 2 and \( k_T = k_R = 9.1 \) for curves in Fig. 3) and non-line-of-sight \( (K = 0) \) conditions between the transmitter and receiver. Parameters

\[
\begin{align*}
\frac{4\beta_{TR}^2}{K + 1} + 8\Delta T_{TR}(\beta_{TR}^2 - \pi^2 - 4\Delta T_{TR}/2\beta_{TR}^2) \\
\frac{4\beta_{TR}^2}{K + 1} + 8\Delta T_{TR}(\beta_{TR}^2 - \pi^2 - 4\Delta T_{TR}/2\beta_{TR}^2) \\
\frac{4\beta_{TR}^2}{K + 1} + 8\Delta T_{TR}(\beta_{TR}^2 - \pi^2 - 4\Delta T_{TR}/2\beta_{TR}^2) \\
\end{align*}
\]

\[
\begin{align*}
\frac{4\beta_{TR}^2}{K + 1} + 8\Delta T_{TR}(\beta_{TR}^2 - \pi^2 - 4\Delta T_{TR}/2\beta_{TR}^2) \\
\frac{4\beta_{TR}^2}{K + 1} + 8\Delta T_{TR}(\beta_{TR}^2 - \pi^2 - 4\Delta T_{TR}/2\beta_{TR}^2) \\
\frac{4\beta_{TR}^2}{K + 1} + 8\Delta T_{TR}(\beta_{TR}^2 - \pi^2 - 4\Delta T_{TR}/2\beta_{TR}^2) \\
\end{align*}
\]
Fig. 3. The theoretical and measured relative power space-delay spectra characteristic for the outdoor M-to-M macro-cell propagation.

Fig. 4. The analytical and measured level crossing rate characteristic for a highway.

used to obtain curves in Figs. 2 and 3 are $\beta_{Tm} = \beta_{Rm} = 15^\circ$, $\theta_T = \theta_R = \pi/4$, $\psi_T = \psi_R = \pi/3$, $\gamma_T = \gamma_R = 0$, $\mu_T = \pi/2$, $\mu_R = 3\pi/2$, $\lambda = 0.3$ m, $R_{11} = R_{12} = 20$ m, $R_{12} = R_{22} = 110$ m, $D = 5$ km, $\gamma = 4$, $L_s = L_c = 2$, and $d_T = d_R \in \{0.1 \lambda, 2 \lambda, 4 \lambda\}$. In Fig. 2, we assume that the single-bounced rays bear more energy ($\theta_T = \gamma_R = 0.45$) than the double-bounced rays ($\gamma_{TR} = 0.1$), which is characteristic for the outdoor M-to-M micro-cell propagation. We can observe that the M-to-M micro-cell psds has two distinct slopes and dies out after 0.6 $\mu$s. In Fig. 3, we consider the multi-cell propagation, i.e. $\theta_T = \gamma_R = 0.2$ and $\gamma_{TR} = 0.6$). In this case, the psds closely follows the one-sided exponential function dies out after 0.8 $\mu$s. Compared to measured relative power delay spectra for SISO system taken from Fig. 2 [8], our theoretical relative power space-delay spectra for $d_T = d_R = 0$ closely match. Finally, Fig. 4 compares the analytical LCR with the measured LCR taken from Fig. 8 of [9]. The measured data are collected on the highway in Germany [9]. The analytical LCR is obtained using the parameters $K = 1.73$, $\mu_T = 31.2^\circ$, $k_T = 18.2$, $\beta_{Tm} = 10^\circ$, $\Delta_T = \Delta_R = 0.6$, $\mu_R = 216.3^\circ$, $k_R = 10.6$, $\beta_{Rm} = 5^\circ$, $\gamma_{TR} = 0.36$, $\eta_T = 0.22$, $\eta_{R} = 0.42$, $D = 300$ m, $\gamma_T = \gamma_R = \Delta_H = 0$, and $f_{Tmax} = f_{Rmax} = 500$ Hz. Fig. 4 shows the close agreement between the theoretical and empirical LCR.

The close agreement between the theoretical and empirical curves in Figs. 2 - 4 confirms the utility of the proposed wideband model.

V. CONCLUSIONS

This paper presented the 3-D theoretical model for wideband MIMO M-to-M channels. Based on this model, the stfcc, the psds, and the LCR are derived for a 3-D non-isotropic scattering environment. Finally, some simulation results are presented and compared with measured data.

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REFERENCES