3-D MIMO Mobile-to-Mobile Channel Simulation

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Abstract—Mobile-to-mobile (M-to-M) radio propagation channels arise in inter-vehicular communications, mobile ad-hoc wireless networks, and relay-based cellular radio networks. The statistical properties of M-to-M channels are quite different from conventional fixed-to-mobile (F-to-M) cellular land mobile radio channels. This article presents a 3-D mathematical reference model for multiple-input multiple-output (MIMO) M-to-M channels. Based on this model, a sum-of-sinusoids based simulation model is proposed for 3-D MIMO M-to-M multipath-fading channels, along with simplified deterministic and statistical models for 2-D non-isotropic scattering. The statistics of the simulation models are verified by simulation.

I. INTRODUCTION

Mobile-to-mobile (M-to-M) communications play an important role in mobile ad-hoc wireless networks, intelligent transportation systems, and relay-based cellular networks. M-to-M communication systems are equipped with low elevation antennas and have both the transmitter ($T_x$) and the receiver ($R_x$) in motion. To successfully design M-to-M systems, it is necessary to have a detailed knowledge of the outdoor multipath fading channel and its statistical properties. Early studies of single-input single-output (SISO) M-to-M Rayleigh fading channels have been reported by Akki & Haber in [1], [2]. They showed that the received envelope of M-to-M channels is Rayleigh faded under non line-of-sight conditions, but the statistical properties differ from fixed-to-mobile (F-to-M) channels. They also proposed a reference model for SISO M-to-M Rayleigh fading channels. Simulation models for SISO M-to-M channels have been proposed in [3]-[5]. Recently, the reference models for narrowband multiple-input multiple-output (MIMO) M-to-M channels have been proposed [6], [7], and corresponding simulation models in [8], [9].

All previously reported models assume that the field incident on the $T_x$ or the $R_x$ antenna is composed of a number of waves travelling only in the horizontal plane. This assumption is acceptable only for certain environments, e.g., rural areas. However, it does not seem appropriate for an urban environment where the $T_x$ and $R_x$ antenna arrays are often located in close proximity to, and lower than, the surrounding buildings. Scattered waves may propagate by diffraction from the edges of buildings down to the street and, thus, not necessarily travel horizontally. To overcome these shortcomings, this paper proposes a three-dimensional (3-D) reference model for MIMO M-to-M multipath fading channels. To describe our 3-D reference model, we first introduce a 3-D geometrical model for MIMO M-to-M channels, referred to as a “two-cylinder” model. This model is extension of the one-cylinder model for F-to-M channels proposed in [10], [11]. By taking into account local scattering around both the $T_x$ and $R_x$, and by including mobility of both the $T_x$ and $R_x$, we obtain our two-cylinder model. The reference model assumes an infinite number of scatterers, which prevents practical implementation. Hence, we propose deterministic and statistical SoS simulation models for a 3-D non-isotropic scattering environment. The statistical properties of our model are derived and verified by simulations.

The remainder of this paper is organized as follows. Section II introduces a geometrical two-cylinder model. Section III presents a 3-D reference model for MIMO M-to-M channels. Section V presents the deterministic and statistical 3-D SoS simulation models along with some representative simulation results. Finally, Section VI provides some concluding remarks.

II. A GEOMETRICAL TWO-CYLINDER MODEL

In this section, we introduce a 3-D geometrical model for MIMO M-to-M channels, called a two-cylinder model. The two-cylinder model is an extension of the one-cylinder model for F-to-M channels proposed in [10], [11]. We consider a narrow-band MIMO communication system with $L_t$ transmit and $L_r$ receive omnidirectional antenna elements. It is assumed that both the $T_x$ and $R_x$ are in motion and equipped with low elevation antennas. The radio propagation environment is characterized by 3-D scattering with non-line-of-sight (NLoS) propagation conditions between the $T_x$ and $R_x$. By taking into account local scattering around both the $T_x$ and $R_x$ and by including mobility of both the $T_x$ and $R_x$, we obtain our two-cylinder model.

Fig. 1 shows the two-cylinder model for a MIMO M-to-M channel with $L_t = L_r = 2$ antenna elements. The two-cylinder model defines two cylinders, one around the $T_x$ and another around the $R_x$, as shown in Fig. 1. Around the transmitter, $M$ fixed omnidirectional scatterers lie on a surface of a cylinder of radius $R_t$, and the $m^{th}$ transmit scatterer is denoted by $S_T^{(m)}$. Similarly, around the receiver, $N$ fixed omnidirectional scatterers lie on the surface of a cylinder of radius $R_r$, and the $n^{th}$ receive scatterer is denoted by $S_R^{(n)}$. The distance between the centers of the $T_x$ and $R_x$ cylinders is $D$. It is assumed that the radii $R_t$ and $R_r$ are much smaller than the distance $D$, i.e., $\max\{R_t, R_r\} \ll D$. The spacing between antenna elements at the $T_x$ and $R_x$ is denoted by $d_T$ and $d_R$, respectively. It is assumed that $d_T$ and $d_R$ are much smaller than the radii $R_t$ and $R_r$, i.e., $\max\{d_T, d_R\} \ll \min\{R_t, R_r\}$. Angles $\theta_T$ and $\theta_R$ describe the orientation of the $T_x$ and $R_x$. 

antenna array in the $x$-$y$ plane, respectively, relative to the $y$-axis. Similarly, angles $\psi_T$ and $\psi_R$ describe the elevation of the $T_x$’s antenna array and the $R_x$’s antenna array relative to the $x$-$y$ plane, respectively. The $T_x$ and $R_x$ are moving with speeds $v_T$ and $v_R$ in directions described by angles $\gamma_T$ and $\gamma_R$, respectively. The symbols $\alpha_T^{(m)}$ and $\alpha_R^{(n)}$ denote the azimuth angle of departure (A AoD) and the azimuth angle of arrival (A AoA), respectively. Similarly, the symbols $\beta_T^{(m)}$ and $\beta_R^{(n)}$ denote the elevation angle of departure (E AoD) and the elevation angle of arrival (E AoA), respectively. Finally, the symbols $\epsilon_{pq}$, $\epsilon_{m,n}$, and $\epsilon_{n,q}$ denote distances $A_T^{(p)} - S_T^{(m)}$, $S_T^{(m)} - S_R^{(n)}$, and $S_R^{(n)} - A_R^{(q)}$, respectively, as shown in Fig. 1.

![Fig. 1. The two-cylinder model for MIMO M-to-M channel with $L_t = L_r = 2$ antenna elements.](image)

### III. A 3-D Reference Model for MIMO Mobile-to-Mobile Channels

This section outlines a reference model for MIMO M-to-M multipath fading channels. The starting point is the two-cylinder model shown in Fig. 1. From the two-cylinder model, we observe that the waves from the $T_x$ antenna elements impinge on the scatterers located on the $T_x$ cylinder and scatter from the scatterers located on the $R_x$ cylinder before they arrive at the $R_x$ antenna elements. In contrast to cellular F-to-M channels where waves are single-bounced, in M-to-M channels waves are double-bounced.

The MIMO channel is described by an $L_x \times L_t$ matrix $H(t) = [h_{ij}(t)]_{L_x \times L_t}$ of complex low-pass faded envelopes. In the 3-D reference model, the number of local scatterers around the $T_x$ and $R_x$ is infinite. Consequently, the received complex faded envelope of the link $A_T^{(p)} - A_R^{(q)}$ is

$$h_{pq}(t) = \lim_{M,N \to \infty} \frac{1}{\sqrt{MN}} \sum_{m=1}^{M} \sum_{n=1}^{N} e^{-j2\pi(\epsilon_{pm} + \epsilon_{mn} + \epsilon_{nq}) + j\phi_{mn}} e^{j2\pi(f_{T\max} \cos(\alpha_T^{(m)} - \gamma_T) \cos(\beta_T^{(m)} + f_{R\max} \cos(\alpha_R^{(n)} - \gamma_R) \cos(\beta_R^{(n)} - \gamma_R))},$$

where $f_{T\max} = v_T/\lambda$ and $f_{R\max} = v_R/\lambda$ are the maximum Doppler frequencies associated with the $T_x$ and $R_x$, respectively, and $\lambda$ is the carrier wavelength. It is assumed that the angles of departures (A AoDs and E AoDs) and the angles of arrivals (A AoAs and E AoAs) are random variables, and that the angles of departure are independent from the angles of arrival. Additionally, it is assumed that the phases $\phi_{mn}$ are random variables uniformly distributed on the interval $[-\pi, \pi]$ and independent from the angles of departure and the angles of arrival.

The distances $\epsilon_{pq}$, $\epsilon_{m,n}$, and $\epsilon_{n,q}$ can be expressed as functions of the random variables $\alpha_T^{(m)}$, $\alpha_R^{(n)}$, $\beta_T^{(m)}$, and $\beta_R^{(n)}$ as follows:

$$\epsilon_{pm} \approx R_t - \frac{d_T}{2} \sin \psi_T \sin \beta_T^{(m)} - \frac{d_T}{2} \cos \psi_T \cos \beta_T^{(m)},$$

$$\epsilon_{pq} \approx R_r - \frac{d_R}{2} \sin \psi_R \sin \beta_R^{(n)} - \frac{d_R}{2} \cos \psi_R \cos \beta_R^{(n)},$$

$$\epsilon_{mn} \approx D.$$ (4)

Derivations of (2) - (4) are omitted for brevity.

Using (2) - (4), the complex faded envelope of the link $A_T^{(p)} - A_R^{(q)}$ can be rewritten as

$$h_{pq}(t) = \lim_{M,N \to \infty} \frac{1}{\sqrt{MN}} \sum_{m=1}^{M} \sum_{n=1}^{N} a_{p,m} b_{n,q} e^{j\phi_{mn} + j\phi_0}$$

$$e^{j2\pi(f_{T\max} \cos(\alpha_T^{(m)} - \gamma_T) \cos(\beta_T^{(m)} + f_{R\max} \cos(\alpha_R^{(n)} - \gamma_R) \cos(\beta_R^{(n)} - \gamma_R))},$$

where $\phi_0 = -2\pi (R_t + R_r + D)/\lambda$ and parameters $a_{p,m}$ and $b_{n,q}$ are defined as

$$a_{p,m} = e^{j2\pi d_T \sin \psi_T \sin \beta_T^{(m)}},$$

$$b_{n,q} = e^{j2\pi d_R \sin \psi_R \sin \beta_R^{(n)}}$$

Note that the constant phase $\phi_0$ can be set to zero without loss of generality because it does not affect statistical properties of the model.

### IV. Space-Time Correlation Function of the 3-D Reference Model

The normalized space-time correlation function between two complex faded envelopes $h_{pq}(t)$ and $h_{\tilde{p}q}(t)$ is

$$R_{pq,\tilde{p}q}[d_R, \tau] \triangleq \frac{E[h_{pq}(t)h_{\tilde{p}q}(t + \tau)]}{\sqrt{E[h_{pq}(t)]^2 E[h_{\tilde{p}q}(t)]^2}},$$

where $(\cdot)^*$ denotes complex conjugate operation, $E[\cdot]$ is the statistical expectation operator, $p, \tilde{p} \in \{1, \ldots, L_t\}$, and
where parameters $x$, $y$, $z$, and $w$ are
\[ x = j2\pi d_{T_y}/\lambda - j2\pi f_{T_{\text{max}}} \cos \gamma_T + k_t \cos \mu_T, \]
\[ y = j2\pi d_{T_y}/\lambda - j2\pi f_{T_{\text{max}}} \sin \gamma_T + k_t \sin \mu_T, \]
\[ z = j2\pi d_{R_x}/\lambda - j2\pi f_{R_{\text{max}}} \cos \gamma_R + k_t \cos \mu_R, \]
\[ w = j2\pi d_{R_x}/\lambda - j2\pi f_{R_{\text{max}}} \sin \gamma_R + k_t \sin \mu_R, \]
and $d_{T_y} = d_T \cos \theta_T \cos \psi_T$, $d_{T_x} = d_T \sin \theta_T \cos \psi_T$, $d_{R_x} = d_R \cos \theta_R \cos \psi_R$, and $d_{R_y} = d_R \sin \theta_R \cos \psi_R$.

Many existing correlation functions are special cases of the 3-D MIMO M-to-M space-time correlation function in (13). The simplest special case of (13) is Clark’s temporal correlation function $J_0(2\pi f_{T_{\text{max}}})$ [15], obtained for $k_T = \beta_T = 0$ (2-D isotropic scattering around $R_x$), $f_{T_{\text{max}}} = k_T = 0$ (stationary $T_x$, no scattering), and $d_T = d_R = 0$ (single-antenna $T_x$ and $R_x$), where $J_0(\cdot)$ is the first kind zeroth-order Bessel function. Expressions for other space-time correlation functions based on the “one-ring” model [16], [17], and based on the “one-cylinder” model [11], [18] can be similarly obtained.

The temporal correlation function for M-to-M channels, assuming 2-D isotropic scattering, $J_0(2\pi f_{T_{\text{max}}})J_0(2\pi f_{R_{\text{max}}})$ [1], is obtained for $k_T = k_R = 0$, $d_T = d_R = 0$, and $\beta_T = \beta_R = 0$. Finally, the space-time correlation function for M-to-M channels, assuming 2-D non-isotropic scattering, $I_0(\sqrt{x^2 + y^2})I_0(\sqrt{z^2 + w^2})/(I_0(k_T)I_0(k_R))$ [7] is obtained for $\beta_T = \beta_R = 0$.

V. SUM-OF-SINUSOIDS SIMULATION MODELS

The reference model described in Section III assumes an infinite number of scatterers, which prevents practical implementation. Here, we propose a simulation model having a finite number of scatterers that closely matches the statistical properties of the reference model.

Using the reference model and assuming 3-D non-isotropic scattering, the following function is considered for the received complex faded envelope:
\[ h_{pq}(t) = \frac{1}{\sqrt{MN}} \sum_{m=1}^{M} \sum_{n=1}^{N} a_{p,m} b_{n,q} \exp \{j\phi_{mn} \} \]
\[ \times \exp \left\{ j2\pi t \left[ f_{T_{\text{max}}} \cos (\alpha_T^n - \gamma_T^n) \cos (\beta_T^n) + f_{R_{\text{max}}} \cos (\alpha_R^n - \gamma_R^n) \cos (\beta_R^n) \right] \right\} , \]
where parameters $x$, $y$, $z$, and $w$ are
\[ x = j2\pi d_{T_x}/\lambda - j2\pi f_{T_{\text{max}}} \cos \gamma_T + k_t \cos \mu_T, \]
\[ y = j2\pi d_{T_x}/\lambda - j2\pi f_{T_{\text{max}}} \sin \gamma_T + k_t \sin \mu_T, \]
\[ z = j2\pi d_{R_x}/\lambda - j2\pi f_{R_{\text{max}}} \cos \gamma_R + k_t \cos \mu_R, \]
\[ w = j2\pi d_{R_x}/\lambda - j2\pi f_{R_{\text{max}}} \sin \gamma_R + k_t \sin \mu_R, \]
\[
\alpha_T^{(m)} = F_T^{-1}(\eta_m), \quad \alpha_R^{(n)} = F_R^{-1}(\delta_n), \quad (16)
\]
for \( m = 1, \ldots, M \), \( n = 1, \ldots, N \). Function \( F_T^{-1}(\cdot) \) denotes the inverse function of the von Mises cumulative distribution function (cdf) and can be evaluated using method in [19]. Parameters \( \eta_m \) and \( \delta_n \) are independent random variables uniformly distributed on the interval \((0, 1)\). The EAOs, \( \beta_T^{(m)} \), and the EAOs, \( \beta_R^{(n)} \), are modelled using the pdfs \( f(\beta_T) = \pi \cos(\pi \beta_T/(2\beta_{\text{max}}))/(4\beta_{\text{max}}) \) and \( f(\beta_R) = \pi \cos(\pi \beta_R/(2\beta_{\text{max}}))/(4\beta_{\text{max}}) \), respectively, and are generated as follows:
\[
\beta_T^{(m)} = \frac{2\beta_T}{\pi} \arcsin(2\nu_m - 1), \quad (17)
\]
\[
\beta_R^{(n)} = \frac{2\beta_R}{\pi} \arcsin(2\zeta_n - 1), \quad (18)
\]
for \( m = 1, \ldots, M \), \( n = 1, \ldots, N \), where parameters \( \nu_m \) and \( \zeta_n \) are independent random variables uniformly distributed on the interval \((0, 1)\).

For the maximum elevation angles \( \beta_T^{(m)} \) and \( \beta_R^{(n)} \) in the range \( 1^\circ \leq |\beta_T^{(m)}, \beta_R^{(n)}| \leq 20^\circ \), all random variables in (15), i.e., \( \alpha_T^{(m)} \), \( \alpha_R^{(n)} \), \( \beta_T^{(m)} \), \( \beta_R^{(n)} \), and \( \phi_{m,n} \), can be treated as mutually independent random variables and the elevation angles can be approximated using \( \cos \beta_T^{(m)} \cos \beta_R^{(n)} \approx 1 \), \( \sin \beta_T^{(m)} \approx \beta_T^{(m)} \), and \( \sin \beta_R^{(n)} \approx \beta_R^{(n)} \). Then, the complex faded envelope in (15) can be approximated as
\[
\alpha_T^{(n)} = R^{-1} \left( \frac{n - 0.5}{N_A} \right), \quad (23)
\]
\[
\beta_T^{(i)} = \frac{2\beta_{Tm}}{\pi} \arcsin \left( \frac{2i - 1}{M_E} - 1 \right), \quad (24)
\]
\[
\beta_R^{(k)} = \frac{2\beta_{Rm}}{\pi} \arcsin \left( \frac{2k - 1}{N_E} - 1 \right), \quad (25)
\]
for \( m = 1, \ldots, M_A \), \( n = 1, \ldots, N_A \), \( i = 1, \ldots, M_E \), \( k = 1, \ldots, N_E \), respectively. The function \( F_T^{-1}(\cdot) \) is the inverse function of the von Mises cdf and is evaluated using method in [19].

For \( M, N \to \infty \), our deterministic model can be shown to exhibit properties of the reference model [20]. The space-time correlation function of the complex faded envelope in (19) matches the approximated space-time correlation function in (13).

Fig. 2 shows the space-time correlation function \( (d_T = d_R = 1\lambda) \) of the deterministic model for a linear antenna array with \( L_A = L_R = 2 \) antennas, using \( M_A = 45, M_E = 5, N_A = 45, \) and \( N_E = 5 \) scatterers. Other parameters used to obtain the curves in Fig. 2 are \( \theta_T = \theta_R = \pi/3, \psi_T = \psi_R = \pi/4, \gamma_T = \pi/6, \gamma_R = \pi/12, \beta_{Tm} = \beta_{Rm} = 15^\circ, k_T = k_R = 2, \) and \( \mu_T = \mu_R = \pi/4 \). Results show that the space-time correlation function of the deterministic model closely matches the theoretical one in the range of normalized time delays, \( 0 \leq f_{T_{\text{max}}} T_s \leq 4 \).

By allowing phases and Doppler frequencies to be random variables, our deterministic model can be modified to match statistical properties of the reference model over a wider range of normalized time delays, while at the same time requiring a smaller number of scatterers. The statistical properties of this (statistical) model vary for each simulation trial, but will converge to desired ensemble averaged properties when averaged over a sufficient number of simulation trials. The complex faded envelope is defined in (19) and the AAODs, the
AAoAs, the EAOs, and the EAoAs are generated as follows:

$\alpha_{T}^{(m)} = F_{T}^{-1}\left(\frac{m + \theta_A - 1}{M_A}\right)$, \hspace{1cm} (26)

$\alpha_{R}^{(n)} = F_{R}^{-1}\left(\frac{n + \psi_A - 1}{N_A}\right)$, \hspace{1cm} (27)

$\beta_{T}^{(i)} = \frac{2\beta_{T_{n}}}{\pi} \arcsin\left(\frac{2(i + \theta_E - 1)}{M_E}\right)$, \hspace{1cm} (28)

$\beta_{R}^{(k)} = \frac{2\beta_{R_{m}}}{\pi} \arcsin\left(\frac{2(k + \psi_E - 1)}{N_E}\right)$, \hspace{1cm} (29)

for $m = 1, \ldots, M_A$, $n = 1, \ldots, N_A$, $i = 1, \ldots, M_E$, $k = 1, \ldots, N_E$, respectively. The parameters $\theta_A$, $\psi_A$, $\theta_E$, and $\psi_E$ are independent random variables uniformly distributed on the interval $[0, 1]$. The function $F_{T/R}^{-1}(\cdot)$ is the inverse function of the von Mises cdf and is evaluated using method in [19].

Fig. 3 shows the space-time correlation function ($d_T = d_R = 1\lambda$) of the statistical model for a linear antenna array with $L_t = L_r = 2$ antennas, using $M_A = 20$, $M_E = 3$, $N_A = 20$, and $N_E = 3$ scatterers and $N_{stat} = 50$ simulation trials. Other parameters are the same as in Fig. 2. Results show that the space-time correlation function of the statistical model closely matches the theoretical one in the wider range of normalized time delays, $0 \leq f_{T_{\text{max}}} T_s \leq 10$.

![Fig. 3. The normalized space-time correlation function ($d_T = d_R = 1\lambda$) of the complex faded envelope of the statistical and reference models.](image)

VI. CONCLUSIONS

In this paper, a two-cylinder geometrical propagation model is introduced. Based on this geometrical model, a 3-D reference model for MIMO M-to-M fading channels is proposed. From the reference model, a closed-form joint space-time correlation function for 3-D non-isotropic scattering environment can be derived. Many existing correlation functions in the literature are special cases of our 3-D MIMO M-to-M space-time correlation function. Finally, deterministic and statistical SoS simulators are presented and shown to closely match the statistical properties of the reference model.

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REFERENCES


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